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Longitudinal and transverse strength of glass ribbon for plastic reinforcement

S. T. GULATI
Corning Glass Works, Research and Development Laboratory, Corning, New York, USA

The strength measurements for Corning Code 8871 glass ribbon with a rectangular cross-section measuring 0.22 in. x 0.002 in. are presented. Owing to the narrow width of the ribbon, tensile strength measurements were carried out only in the longitudinal direction. To establish the isotropy of ribbon strength, flexure tests were conducted in both the longitudinal and transverse directions. It was found that in the as-manufactured condition, the transverse strength is lower than the longitudinal strength, indicating preferred orientation of flaws introduced during manufacturing. After etching in 10% HF/15% HNO_3 solution for different lengths of time, the longitudinal and transverse flexural strengths increased with etch time, reaching an asymptotic value after 20 sec. The transverse strength value, after etching, was generally higher than the longitudinal strength, partly due to the considerably smaller surface area exposed to tension and partly due to the uncertainties in measurement of chord length. The anisotropy of ribbon strength in the as-manufactured condition could partly account for the anisotropic strength of ribbon-reinforced composites as reported in the literature [4, 5].

1. Introduction

The present paper is concerned with the isotropy of the strength of glass ribbon. As a reinforcement for plastics, the ribbon geometry, due to its large aspect ratio, offers many advantages over fibres. Owing to the sheet-like structure, ribbons provide two-dimensional isotropic stiffening in terms of both elastic and shear moduli [1]. They permit large volume fractions, unattainable by fibres, approaching values as high as 90%, as reported by Humphrey [2]. Unlike fibres, ribbons do not require special orientation to achieve isotropic properties analogous to 0, ± 45°, 90° fibre-reinforced composite. Also in view of the large aspect ratio, ribbon reinforcement provides a tortuous path for permeation of contained fluids and consequently results in a thousand-fold [3] greater permeation resistance than that of a fibre-reinforced composite containing equivalent volume fraction. The strength of ribbon composites, however, has not proven to be isotropic; the longitudinal strength of ribbon composites is generally reported to be twice the transverse strength [4, 5]. Of the the many factors which effect the strength of ribbon composites, the strength of ribbon has a direct bearing on it. It is reasonable to expect isotropic stiffness for the ribbon since stiffness is an intrinsic property. The strength, on the other hand, being an extrinsic property, depends on the surface quality of the ribbon and may or may not be isotropic, depending on the presence or absence of oriented flaws. A significant factor which could explain the anisotropic strength of ribbon composites is the strength anisotropy of ribbon itself.

Flaws with preferred orientation can occur during ribbon manufacture, whether the ribbon is drawn from hot melt or redrawn from a specially prepared blank to provide the desired cross-sectional geometry. In both these processes, the flaws would tend to orient themselves in the longitudinal or draw direction, thereby degrading the transverse strength. It is, therefore, imperative that the strength isotropy, or lack thereof, be established before the composite strength can be analysed. The ribbon used for this study was made of high lead-containing silica glass with a density of 3.84 g cm^{-3} and a nearly rectangular cross-section, which measured 0.22 in. x 002 in.

Usually, it is the tensile strength of ribbon which figures into the computation of composite strength. This can readily be determined by measuring the load at failure, in a direct tensile test, of individual ribbons. However, such a test is possible only in the ribbon direction, since the ribbon is generally too narrow in the width direction to lend itself to direct pull. Thus, flexure tests were devised to measure the strength in the two directions. The longitudinal flexural strength was also compared with the longitudinal tensile strength and was found to be larger, due to the smaller surface area under tension in the flex test. The transverse flexural strength was always lower than the longitudinal flexural strength, indicating presence of oriented flaws. In an attempt to enhance the strength values, the ribbons were etched for different lengths of time in 10% HF/15% HNO_3 solution and the enhancement of strength in the two directions was noted. The longitudinal strength reached a plateau value after an etch time of 20 sec, but the transverse strength continued to increase up to an etch time of 30 sec. This suggests that the flaws affecting the transverse strength were more severe and took longer to etch. The analyses of flexure of thin ribbon together with the accuracy of this method of measuring ribbon strength are discussed.

2. Longitudinal flexural strength

Fig. 1 shows the flexure jig used for bending the ribbon longitudinally. The ribbon, approximately 8 in. long, is affixed by Scotch tape onto the vertical platens of the fixture, one of which is fixed while the other is movable. The motion is imparted to the platen by turning the lead screw. The platen rides in a guide (not shown in Fig. 1), which prevents it from cocking. The distance between the two platens is read off roughly by the scale mounted on the base and is given more precisely by the graduated crankwheel, to an accuracy of 1 mil.* This distance at the instance of ribbon failure is a measure of the radius of curvature R, which is inversely proportional to the strength of ribbon.

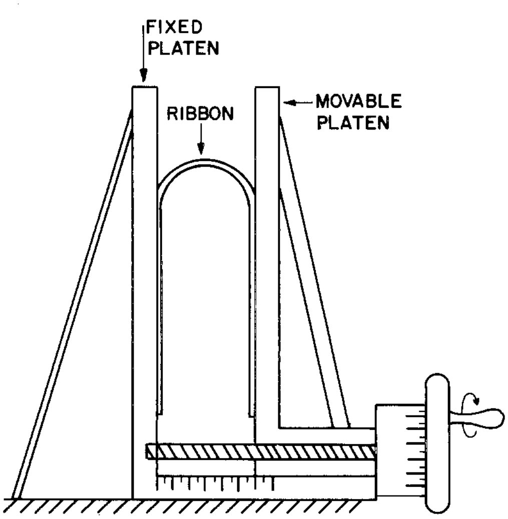

Figure 1 Flexure jig.

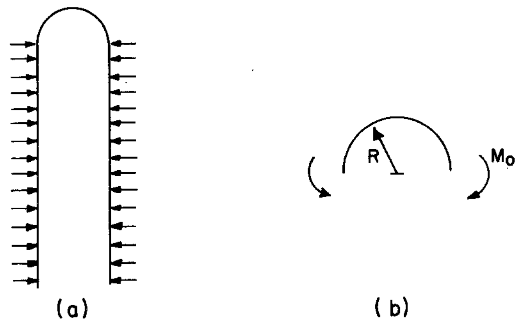

Figure 2 Force analysis of longitudinally flexed ribbon.

The forces exerted on the ribbon by the flexure jig are shown in the free body diagram in Fig. 2a. It has been assumed that a constant force is exerted by the platen all along the ribbon. This force system is equivalent to a moment M_0 and the ribbon deforms to a constant curvature $(1/R)$. The flexural stress, treating the ribbon as a thin plate, is related to the applied moment M_0 by the following equations:

$$\sigma = \frac{12\,Mz}{t^3}, \tag{1}$$

$$M = \frac{D}{R}, \tag{2}$$

$$D = \frac{EI}{b\,(1-\nu^2)}, \tag{3}$$

where E and ν are the elastic modulus and Poisson's ratio of ribbon glass, t and b are its thickness and width, I is the area moment of inertia of the ribbon, and z is the distance measured from the middle surface of the ribbon to the point where stress is desired. The maximum value of stress occurs at the surface of the ribbon where $z = t/2$.

* 1 mil = 2.5400×10^{-5} m.

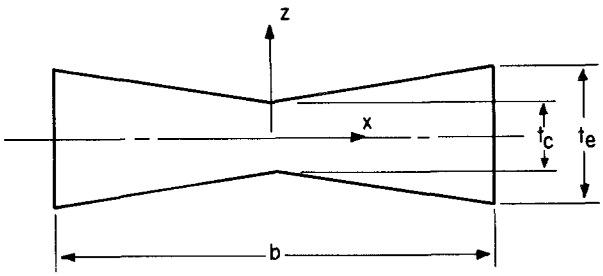

Figure 3 Thickness variation for $\frac{1}{4}$in. redrawn code 8871 ribbon.

The foregoing equations need to be modified to account for the anticlastic curvature and the non-uniformity in the ribbon thickness, as shown in Fig. 3. When the ribbon width is large in comparison with its thickness, such a ribbon upon flexing exhibits curling of the edges, forming an anticlastic surface. The stress given by Equation 1 is modified by a factor k $(k > 1)$, which in turn depends on the ribbon geometry and Poisson's ratio [6]. Following Conway and Nickola [6],

$$k = 1 - \nu^2 + \nu\sqrt{\left(\frac{1-\nu^2}{3}\right)} \tag{4}$$

and

$$\sigma = \frac{12\,kMz}{t^3}. \tag{5}$$

The maximum value of stress occurs at the edge and $z = t/2$.

With regard to thickness variation, the ribbon had a minimum thickness at the midwidth, t_c, which increased to a value t_e at the edge. For ease of computation, a linear variation was assumed for the thickness, i.e.

$$t = t_c + 2\left(\frac{t_e - t_c}{b}\right)x. \tag{6}$$

The exact analysis of anticlastic bending of strips of variable thickness has been given by Conway and Farnham [7] and involves Kelvin functions. However, the following approximate analysis was adopted for our purposes. The area moment of inertia I for the cross-section shown in Fig. 3 is given by

$$I = \frac{b}{12}t_c^3 + 4\left[\frac{1}{2}\cdot\frac{b}{2}\cdot\left(\frac{t_e - t_c}{2}\right)\right]\left[\frac{t_c}{2} + \frac{1}{6}(t_e - t_c)\right]^2$$

$$= \frac{b}{72}(t_e^3 + 3t_c t_e^2 + 2t_c^3). \tag{7}$$

The quantity $1/t^3$ in Equation 1 is averaged over the width by integration, so that

$$\frac{1}{t^3} = \frac{2}{b}\int_0^{b/2}\frac{\mathrm{d}x}{t^3}$$

$$= \frac{2}{b}\int_0^{b/2}\frac{\mathrm{d}x}{\left[t_c + 2\left(\frac{t_e - t_c}{b}\right)x\right]^3}$$

$$= \frac{1}{2}\left(\frac{t_e + t_c}{t_e^2 t_c^2}\right). \tag{8}$$

Substitution of Equations 7 and 8 into Equations 1 to 3, together with $z = t_e/2$, results in the following expression for maximum longitudinal flexural stress in the ribbon:

$$\sigma_{\max}^{L} = \frac{E}{12}\left(\frac{t_e + t_c}{2R}\right)\left\{1 + \nu\sqrt{\left[\frac{1}{3(1-\nu^2)}\right]}\right\}\left(\frac{2\tau^3 + 3\tau + 1}{\tau^2}\right), \tag{9}$$

where τ denotes the thickness ratio t_c/t_e. For Corning Code 8871 glass ribbon $\nu = 0.26$ and we obtain:

$$\sigma_{\max}^{L} = 0.57\,E\left(\frac{t_e + t_c}{2R}\right)\left(\frac{1 + 3\tau + 2\tau^3}{6\tau^2}\right). \tag{10}$$

In the case of uniform thickness,

$$t_e = t_c = t,$$

$$\tau = 1,$$

and Equation 10 reduces to

$$\sigma_{\max}^{L} = 0.57\frac{Et}{R}. \tag{11}$$

Equation 10 was used for all the strength measurements reported here and requires only the knowledge of the radius of curvature, R, at failure. The elastic modulus of Code 8871 glass was taken as 8.4×10^6 psi.*

* 10^3 psi $\equiv 6.89$ N mm^{-2}.

TABLE I Longitudinal flexural strength versus etch time

Etch time (sec)	Thickness variation (mil)	Radius of curvature at failure (mil)	Mean flexural strength (10^3 psi)	Standard deviation (10^3 psi)	95% confidence interval on mean strength (10^3 psi)
0	1.6 – 2.0	71 – 106	111.9	13.7	± 9.8
5	1.4 – 1.9	41 – 56	204.4	21.2	± 15.2
10	1.3 – 1.8	23 – 39	302.9	53.5	± 38.3
20	1.1 – 1.5	25 – 39	254.6	43.8	± 31.3
25	1.0 – 1.5	24 – 45	222.2	34.8	± 24.9
30	0.9 – 1.3	22 – 30	249.8	22.9	± 16.4

Samples of ten ribbons were etched for different lengths of time from 0 to 30 sec. The thickness variation, range of radii of curvature to failure for each set of samples, and the mean flexural strength in the longitudinal direction, are given in Table I. The reduction in ribbon thickness with etch time is plotted in Fig. 4, which shows that a maximum of 0.7 mil thickness is etched away after 30 sec. The strength data are plotted in Fig. 5 as function of etch time. For comparison purposes, the longitudinal tensile strength of ribbons with 4 in. gauge length is also shown. It is clear that both the flexural and tensile strengths achieve an asymptotic value at 20 sec etching time. Also, the flexural strength is higher than the tensile strength, due to the large difference in the surface areas subjected to maximum tensile stress in the two tests. In flexure experiments, the surface area under tension was 10 to 30 times less than that in the tensile tests.

3. Transverse flexural strength

Owing to the extremely small width, the ribbons were not flexed in the transverse direction in the jig shown in Fig. 1. The manner in which the ribbon was flexed transversely is shown in Fig. 6. Short lengths of ribbons, approximately 2 in. long, were partially wrapped around their width by a piece of Scotch tape $\frac{1}{2}$ in. wide. This facilitated subsequent handling and flexing of the sample. The surface to be subjected to tension was left bare. The ribbon was flexed essentially by buckling, which was brought about by pushing on the edges. The deformed shape, in the case of transverse flexure, does not have a constant curvature. It is clear that the bending moment given by

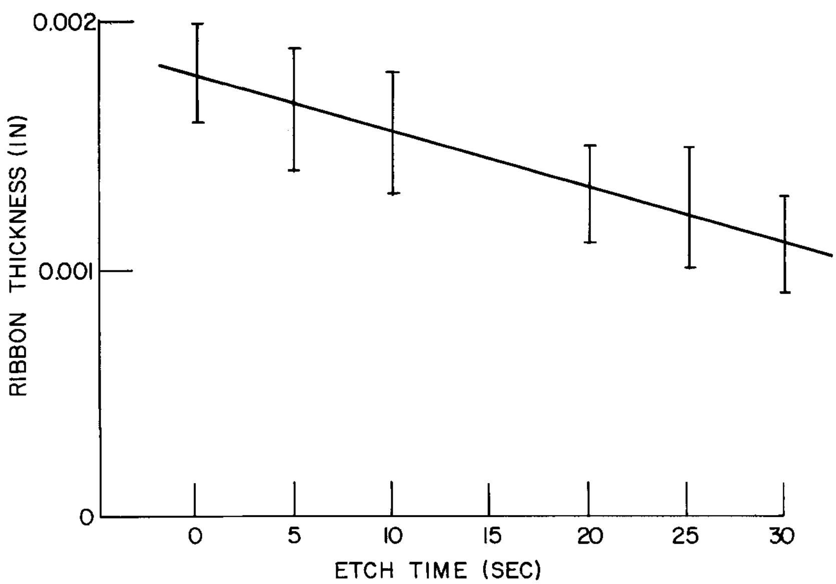

Figure 4 Variation of ribbon thickness with etch time.

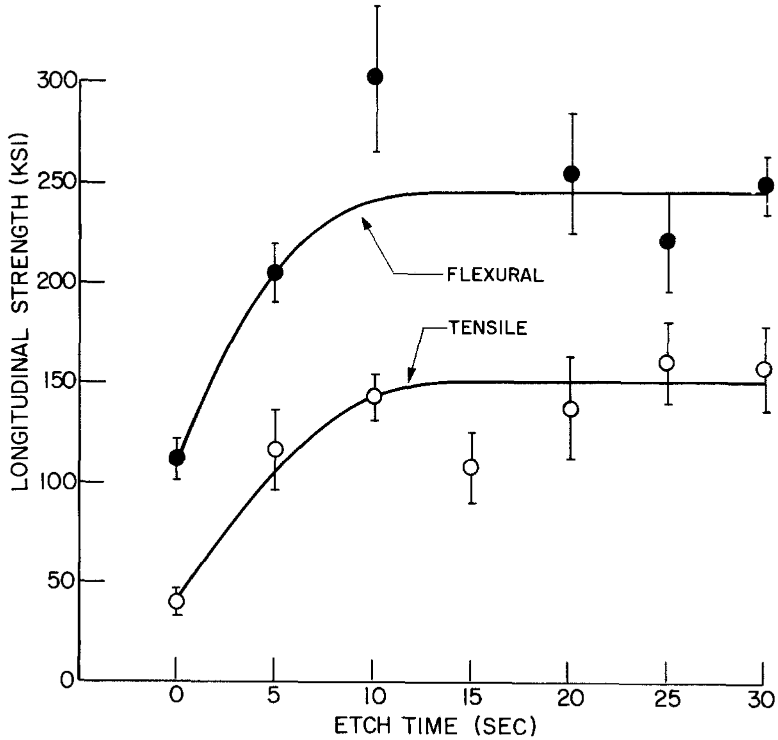

Figure 5 Longitudinal strength versus etch time. 1 ksi = 10^3 psi = 6.89 N mm^{-2}.

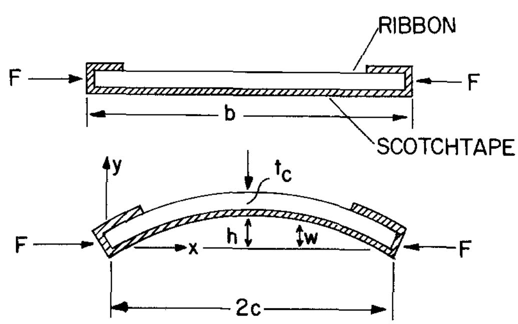

Figure 6 Transverse flexure of ribbon.

$[F \cdot w]$ has a maximum value at midwidth where the deflection w is a maximum. Thus, the radius of curvature has its minimum value at the midwidth. Accordingly, the maximum value of flexural stress occurs at the midwidth and is confined to a small fraction of the surface area.

Guided by the fact that a column buckles into a sine wave, we assume that the ribbon deforms into a sinusoidal form and we compute its radius of curvature taking into account its inextensibility. The equation describing the deformed shape of the ribbon is given by

$$y = h \sin \frac{\pi x}{2c}, \tag{12}$$

where h is the maximum deflection at midwidth and $2c$ denotes the "chord length" or distance between the approaching edges of the ribbon. The arc length of the deformed segment must equal the original width b of the ribbon under the condition of inextensibility, i.e.

$$\begin{aligned} b &= \int_0^{2c} \sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]} dx \\ &= \int_0^{2c} \sqrt{\left[1 + \frac{\pi^2 h^2}{4c^2} \cos^2\left(\frac{\pi x}{2c}\right)\right]} dx. \end{aligned} \tag{13}$$

Introducing the variable

$$\phi = \frac{\pi x}{2c},$$

the above equation assumes the form

$$b = \frac{2c}{\pi} \int_0^{\pi/2} \sqrt{(1 + K^2 \cos^2 \phi)}\, d\phi, \tag{14}$$

with

$$K = \frac{\pi h}{2c}. \tag{15}$$

This may be written as a complete elliptic integral of the second kind, which is tabulated in standard handbooks [8], by replacing $\cos^2 \phi$ in terms of

TABLE II Transverse flexural strength versus etch time

Etch time (sec)	t_c (mil)	$2c$ (mil)	R (mil)	Mean flexural strength (10^3 psi)	Standard deviation (10^3 psi)	95% confidence limit on mean value (10^3 psi)
0	1.6	170 – 200	43 – 132	94.5	47.5	± 34
5	1.4	120 – 150	11 – 24	395	115.6	± 82.7
10	1.3	120 – 160	11 – 24	345.8	107.9	± 77.2
20	1.1	80 – 120	3 – 11	748	392	± 281
25	1.1	100 – 130	6 – 14	532	197.5	± 141
30	0.9	80 – 150	3 – 24	791	471	± 337

$\sin^2\phi$. We obtain

$$\frac{\pi b}{4c\sqrt{(1+K^2)}} = \int_0^{\pi/2} \sqrt{1-\lambda^2\sin^2\phi}\, d\phi, \quad (16)$$

where

$$\lambda^2 = \frac{K^2}{1+K^2} = \frac{\pi^2}{4}\frac{(h/c)^2}{\left[1+\frac{\pi^2}{4}\left(\frac{h}{c}\right)^2\right]} < 1. \quad (17)$$

The integral in Equation 15 is the complete elliptic integral of the second kind and may be expressed in series [8]

$$\int_0^{\pi/2} \sqrt{(1-\lambda^2\sin^2\phi)}\, d\phi = \frac{\pi}{2}\left(1-\frac{\lambda^2}{4}-\frac{3}{64}\lambda^4\ldots\right). \quad (18)$$

Neglecting terms like $\frac{3}{64}\lambda^4$ and all higher order terms* and substituting into Equation 15 and simplifying, we obtain

$$9K^4 + \left\{24 - 16\left(\frac{b}{2c}\right)^2\right\}K^2 - 16\left\{\left(\frac{b}{2c}\right)^2 - 1\right\} = 0, \quad (19)$$

whose solution is given by

$$18K^2 = 16\left(\frac{b}{2c}\right)^2 - 24 + 8\left(\frac{b}{2c}\right)\sqrt{\left[4\left(\frac{b}{2c}\right)^2 - 3\right]}. \quad (20)$$

The radius of curvature, R, is given by

$$\frac{1}{R} = \frac{-y''}{[1+(y')^2]^{3/2}}, \quad (21)$$

where prime denotes differentiation with respect to x. At the point of interest, namely $x = c$, the slope of the deformed shape is zero so that

$$R_{\min} = -\frac{1}{y''}\bigg|_{x=c} = \frac{4c^2}{\pi^2 h} = \frac{h}{K^2} = \frac{2c}{\pi K}. \quad (22)$$

The chord length, $2c$, is measured with a scale and $R_{\min}$ calculated from Equations 20 and 22. In the case of transverse flexure, the anticlastic curvature effect is absent, since the ribbon is long compared with its width. The stress is then related to the radius of curvature through Equations 1 to 3. The maximum value of transverse flexure stress is given by substituting $z = t_c/2$ and $I = bt^3/12$ in these equations so that

$$\sigma_{\max}^{T} = \frac{0.5\, Et_c}{R\,(1-\nu^2)}, \quad (23)$$

where R is given by Equation 22.

Ten specimens of ribbon were flexed transversely for each etch time. The results are shown in Table II. Strengths as high as 800 000 psi were calculated using above equations and the measured value of chord length. As mentioned previously, the surface subjected to maximum stress during transverse flexure is considerably smaller than that in the case of longitudinal flexure. Secondly, there are some inaccuracies in the measurement of chord length, measured value being lower than the true value. It was found that a 10% error in $2c$ would affect the stress by 40%. Thus the true value of transverse flexural strength is lower and may be equal to the longitudinal flexural strength, particularly after all the flaws are removed by etching.

The results in Table II are plotted in Fig. 7 together with the longitudinal strength for comparison purposes. It is seen that in the unetched condition the transverse strength is lower than the

* The error introduced as a result of this approximation is less than 10%.

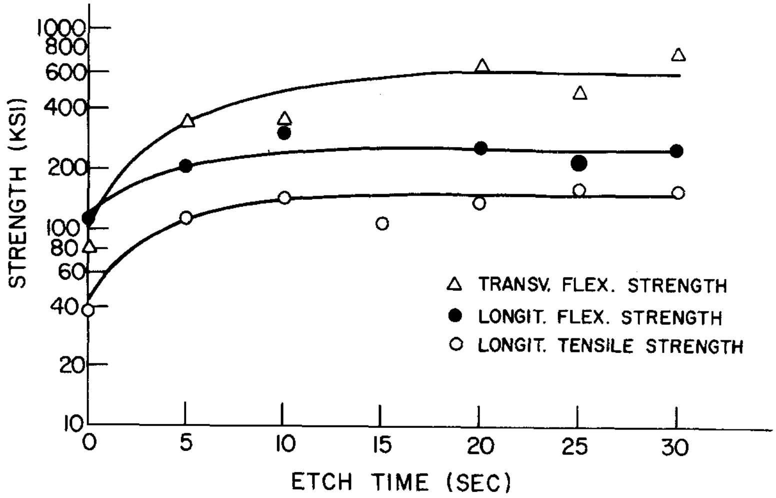

Figure 7 Ribbon strength versus etch time. 1 ksi = 10^3 psi = 6.89 N mm^{-2}.

longitudinal flexural strength, indicating the strength anisotropy due to manufacturing flaws. The fracture pattern observed with this mode of flexure is quite interesting and is shown in Fig. 8. The pattern is completely symmetrical about the midwidth and midlength. The crack density was found to increase with the ribbon strength, indicating the release of stored elastic energy upon fracture. The curving of the cracks also indicates the stress decay with distance from load application point.

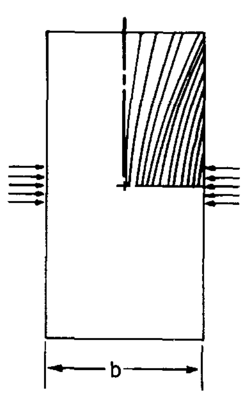

Figure 8 Fracture pattern in transverse flexure.

4. Conclusions

Although the method for measuring the transverse flexural strength is not as accurate as that for the longitudinal flexural strength, the results of these tests clearly point out that the ribbon strength is not isotropic in the as-drawn state.* With etching, the ribbon strength is enhanced considerably and should be isotropic, although this is difficult to ascertain due to unequal surface areas being exposed to the maximum stress.

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* The transverse strength results suggest the need for a sample size larger than 10 and a more accurate method of measuring the chord length to reduce the standard deviation.